Glyph-Based Comparative Stress Tensor Visualization in Cerebral Aneurysms

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Abstract
We present the first visualization tool that enables a comparative depiction of structural stress tensor data for vessel walls of cerebral aneurysms. Such aneurysms bear the risk of rupture, whereas their treatment also carries considerable risks for the patient. Medical researchers emphasize the importance of analyzing the interaction of morphological and hemodynamic information for the patient-specific rupture risk evaluation and treatment analysis. Tensor data such as the stress inside the aneurysm walls characterizes the interplay between the morphology and blood flow and seems to be an important rupture-prone criterion. We use different glyph-based techniques to depict local stress tensors simultaneously and compare their applicability to cerebral aneurysms in a user study. We thus offer medical researchers an effective visual exploration tool to assess the aneurysm rupture risk. We developed a GPU-based implementation of our techniques with a flexible interactive data exploration mechanism. Our depictions are designed in collaboration with domain experts, and we provide details about the evaluation.

Categories and Subject Descriptors (according to ACM CCS): I.4.9 [Computer Graphics]: Image Processing and Computer Vision—Applications

1. Introduction
Glyph-based visualizations enable an efficient and comparative exploration of complex medical data \cite{VPG14}. One of their applications is the depiction of diffusion tensor imaging (DTI) data to show the anatomical structures of fibrous tissues. Tensor data play also an important role in the investigation of vascular diseases, such as cerebral aneurysms. These are pathological dilatations of the vessel wall located at intracranial arteries. Their most serious consequence is their rupture, which leads to a subarachnoid hemorrhage and is associated with a high mortality and morbidity rate.

Besides morphological and hemodynamic factors, structural stresses in the vessel walls influence the aneurysm evolution and rupture risk. Computational Fluid Dynamic (CFD) simulations and Fluid-Structure Interaction (FSI) enable the investigation of the patient-specific wall mechanics and hemodynamics. However, the obtained data are very complex. Besides scalar and vectorial parameters such as Wall Shear Stress (WSS) or internal blood flow, tensor data are calculated that describe stresses within the aneurysm walls. Experts are interested in correlations between hemodynamic factors that are associated with an increased risk of rupture. Therefore, visualization tools are essential that are capable of presenting these different information in an integrated view.

Current visualizations enable a simultaneous depiction of scalar parameters on the aneurysm wall, including wall thickness, wall deformation, and blood flow \cite{MVB17}. However, the analysis of structural stresses without a reduction of the tensor data to a scalar field was not possible so far. We present the first integrated visualization of stress tensors as well as scalar and vectorial parameters based on CFD coupled with FSI. For the tensor depiction, we evaluate four glyph-based techniques, see Fig 1. These methods enable a comparative visualization of tensor data between the inner and outer vessel wall, which is important to localize rupture-prone regions. Moreover, the suitability of the glyphs to perform specific tasks was evaluated in a user study with 60 participants. To support the time-dependent data analysis, optimized views on the aneurysm surface are calculated based on the input data such as stress magnitude or wall thickness. In summary, our main contributions are:

- A comparison of four glyph-based techniques to depict local stress tensors on the aneurysm walls during the cardiac cycle.
- A 3D view of the aneurysm surface that enables a simultaneous exploration of rupture-prone scalar, vectorial and tensor data.
- An automatic selection of viewpoints to support the exploration of the time-dependent aneurysm data.
2. Related Work

Related work comprises comparative and glyph-based depictions within a medical context, and the exploration of aneurysm data.

2.1. Glyph-based Medical Visualization

Glyphs allow an effective depiction of several parameters by encoding them into different properties, e.g., color, size and shape [ZCH'17]. General glyph design guidelines were proposed by Borgo et al. [BKC'13]. Ropinski et al. [ROP11] presented a taxonomy and guidelines for glyph-based medical visualizations. Thereby, two applications were focused: glyphs for DTI and cardiac data. We summarize approaches for visualizing DTI data that is strongly related to the depiction of aneurysm stress tensors.

Diffusion tensors, represented as a $3 \times 3$ positive-definite symmetric matrix, are mostly illustrated as an ellipsoid, where the corresponding eigenvalues and eigenvectors are mapped to its shape and orientation [PB96]. Sigfridsson et al. [SEHW02] presented an approach for visualizing tensor data by combining direct volume rendering (DVR) of a scalar field and glyphs. While DVR serves as an overview depiction, they used ellipsoids and line-based glyphs to get detailed insights into the data. Westin et al. [WME'99] proposed a glyph combining a sphere, a disk and a rod to directly show the linear, planar, and spherical components. However, ellipsoids cause problems of ambiguity. Thus, Kindlmann [Kin04] introduced superquadric (SQ) glyphs. These glyphs improve the visual perception as confirmed by Jankun-Kelly et al. [JKLS10]. Schultz and Kindlmann [SK10] extended the SQs for general symmetric tensors. Hlawitschka et al. [HES08] used a GPU ray-casting technique to render a large amount of SQs in interactive frame rates.

2.2. Visualization and Exploration of Aneurysms

Blood flow data are highly important in order to understand the progression of cerebrovascular diseases, such as aneurysms. Preim and Botha [PB13] provide a summary about the visualization of simulated and measured flow data.

Tomandl et al. [THB'03] developed a DVR approach to investigate the aneurysm morphology that supports diagnosis and intervention planning. For a simultaneous analysis of morphological and hemodynamic data, visualizations are needed that address clutter and occlusion problems. Gasteiger et al. [GNBP11] introduced the FlowLens, to explore anatomical and vectorial hemodynamic data simultaneously. However, occlusion problems occur, because blood flow outside the lens cannot be observed. Thus, Lawonn et al. [LGP14] provided a vessel visualization where the blood flow is always visible. They provide no information about the wall thickness and wall deformation that are important rupture risk factors [ODCP'07]. Gläßer et al. [GLH'14] enabled the visual exploration of vessels with wall thickness without a depiction of the blood flow. Lawonn et al. [LGV'16] presented an occlusion-free blood flow visualization combined with wall thickness by using illustrative techniques. Meuschke et al. [MV'17] presented an integrated vessel visualization considering wall thickness and deformation as well as hemodynamic parameters and internal blood flow. They enabled the simultaneous exploration of scalar properties, but a comparative visualization of tensor data is still missing.

2.3. Comparative Visualizations

Comparative visualizations facilitate the analysis of stress tensors on the inner and outer vessel. Gleicher et al. [GAW'11] proposed a taxonomy with three major comparative designs: juxtaposition, superposition, and explicit encoding. Juxtaposition shows objects side-by-side, such as the blood flow depiction by Angelelli et al. [AH11]. It is effective if comparison is performed within an eyespan. Kölesár et al. [KBVH17] introduced a more general approach to generate comparative visualizations. Superposition is an overlay of different objects, whereas explicit encoding displays relationships between objects. Both techniques are more appealing for the complex aneurysm data. Van Pelt et al. [vPGL'14] presented a comparative depiction for evaluating various stent configurations, integrating the vessel morphology and hemodynamic. Similar to our work, Zhang et al. [ZSL'16] used different SQ glyphs to visualize local differences between two diffusion tensors by combining juxtaposition and explicit encoding. However, their approach is based on normalized tensor data, which is not possible for our data. Therefore, we designed other comparative tensor visualizations.

3. Medical and Hemodynamic Background

In clinical routine, several morphological aneurysm features are used to assess the rupture risk [LEBB09]. These parameters dif-

Figure 1: Comparative visualization of aneurysm stress tensors by using four glyph-based techniques. In (a) optimal views on the aneurysm surface are shown that support the data exploration. In (b) superquadrics and in (c) kite-shaped glyphs show the local tensor on the inner (yellow) and outer wall (blue). In (d), streamlines indicate the tensor main direction for the inner (yellow, purple) and outer wall (blue, green). Scatterplots (e) show the distribution of the main directions within a surface region.
fer statistically significantly between ruptured and unruptured aneurysms [BVV’14]. However, the patient-specific rupture risk cannot be assessed accurately using these criteria. CFD allows a detailed description of intra-aneurysmal hemodynamics [JBS’15], which plays a major role in the growth and rupture risk [CMWP11]. Despite several studies related to rupture-prone flow properties, local WSS is an example that the interpretation is rather difficult and conflicting [MTXS13]. The event of rupture can be defined as the moment when local wall stress exceeds wall strength. FSI provides insights into the stress state of the wall by capturing the interaction of internal wall mechanics and blood flow. This local stress is a tensor containing six independent values that complicates its interpretation and analysis. Therefore, stress tensor visualizations are needed to transfer findings from FSI to clinical discussions.

4. Data Acquisition and Preprocessing

In this section, we describe the data acquisition pipeline that consists of four steps, illustrated in Fig. 2.

Acquisition. First, clinical image data including CT angiography, MR angiography and 3D rotational angiography of the aneurysm morphology are acquired. Common clinical resolutions are up to $512 \times 512 \times 140$ with a voxel size of $0.35 \times 0.35 \times 0.9$ mm.

Extraction of the 3D Vessel Mesh. Based on the image data, the vessel mesh is reconstructed using the pipeline by Mönch et al. [MNP11]. First, a threshold-based segmentation, followed by a connected component analysis is applied to separate the aneurysm and its parent vessel from the surrounding tissue, then Marching Cubes yields the 3D vessel mesh. For the preparation of a geometric model for FSI, it is necessary to manually correct artifacts [MNP11]. Moreover, the mesh quality was optimized by a combination of metric and topological changes [Sch97].

Fluid-Structure Simulation. Hemodynamics inside the fluid domain is numerically solved using CFD based on the finite volume discretization of the Navier-Stokes Equations [SAC’13]. The wall behavior is discretized using the finite element method and local strains and stresses are calculated numerically based on the conservation of momentum. Both domains are handled in a segregated manner. The interaction takes place at the interface of both domains and is performed between the time steps by exchanging shear forces and pressure as well as wall deformation and velocity, respectively.

Simulation specification. Blood is assumed as an incompressible Newtonian fluid with a density of 1055 kg/m$^3$ and a dynamic viscosity of 0.004 Pa s [SAC’13]. For two cases, subject-specific velocity profiles are available [BRB’15], which were used as inlet boundary condition. The deformable wall is considered as homogeneous, isotropic and linear elastic. The Young’s modulus (1 MPa) and Poisson’s ratio (0.45) are taken from [VLR’08]. Wall thickness is assumed to have a major impact on the local stresses [VGH’16]. However, such data cannot be extracted from radiologic image data, because thickness of cerebral walls is below the spatial image resolution. Thus, thickness is set in the range of 0.2 to 0.6 mm obtained by normal extrusion of the wall [VLR’08]. To prevent rigid body motion, the vessel in- and outlet cross-sectional planes are fixed. Other domain parts have no motion restrictions. The domain size of the fluid and the solid are between 237 to 369 $K$ and 55 to 138 $K$ finite element cells, respectively. Solvers were STAR-CCM+ for the fluid domain and Abaqus FEA for the solid domain. Time steps are set to 0.001 s (fluid) or maximally 0.01 s (solid), while the coupling takes place every 0.01 s. Overall calculation times on a standard workstation, using four Intel Xeon E3 cores with 3.5 GHz and 32 GB RAM, varied between 4 and 16 hours.

Concept of mechanical stress. Mechanical stress is a physical quantity caused by mechanical deformation. The stress state of an infinitesimal element is described by the stress tensor $T$ with $3 \times 3$ components. According to the conservation of angular momentum, the stress tensor is symmetric resulting in six independent components. Rotating the tensor leads to zero shear stress and maximum normal stress components, which are obtained by the eigenvectors. Regarding aneurysms, stress inside the wall is caused by the hemodynamic pressure and leads mainly to tension in circumferential direction. Accordingly, the first two eigenvectors are expected to be tangential and the third to be normal to the wall, respectively. Due to the loading condition and the thin aneurysm wall, some studies, e.g., [SAC’13] calculate the wall stress based on shells instead of solid elements. As a consequence, some effects in normal direction of the wall which are less dominant are neglected.

5. Requirement Analysis

There are several requirements that should be met for the visualization of stress tensors in cerebral aneurysms. Our approach is based on discussions with two domain experts: one neuroradiologist with 16 years of work experience and one CFD engineer working on FSI simulations for cerebral aneurysms with three years of work experience. Additionally, the neuroradiologist treats cerebral aneurysms regularly and is involved in clinical research.

The aim of the experts is to better understand the rupture risk. They are trying to find parameter correlations that might have caused the rupture for a patient-specific risk assessment in the future. The typical workflow is to examine the blood flow and induced
measurements on the vessel surface, e.g., WSS or pressure. However, the experts consider the exploration of scalar stress values as not sufficient, since rupture-relevant tensor data are neglected. Thus, a visualization of local stress tensors using the common stress theory of von Mises (see [VLR’08]) is needed. This is a challenging task, as it comprises three directions with corresponding stress values that should be depicted. Moreover, the stress tensors are calculated on the inner and outer vessel wall and the experts would prefer a simultaneous depiction of both tensors to compare their components. It would be possible to calculate differences between the walls for each component, resulting in three scalar fields for the stress values and directions, respectively. Each scalar field could be color-coded on the surface. To correlate differences, the depictions could be juxtaposed. Then, the user would have to mentally combine multiple views, which is quite challenging. Furthermore, the experts are interested in exploring changes of the tensor directions over time and along the aneurysm geometry. In addition, a combined visualization of all quantities would allow to localize foci of dangerous activity (e.g., where is the stress field homogeneous across the wall thickness and where not). This includes the analysis of the tensor components within a conspicuous region. Regions with a homogeneous stress field exhibit only slight changes of the tensor components. Our experts assumed that thin wall regions with a homogeneous tensor field are less rupture prone than thin wall areas with inhomogeneous tensor data. Finally, the manual effort for the time-dependent data exploration should be reduced, since this can be a time-consuming and tedious process depending on the morphological complexity. Based on these discussions and on literature search, we summarize the main requirements for such a framework as follows:

Req. 1. It should show the main directions of the stress tensors during the animation.

Req. 2. The corresponding stress values along the main directions should be depicted.

Req. 3. A comparative depiction of local stress tensors on the inner and outer vessel wall is needed.

Req. 4. A visualization is needed that shows the tensor quantities with simultaneous depiction of scalar and vectorial data.

Req. 5. A visualization is needed that shows the distribution of the stress tensor directions within a surface region.

Req. 6. An automatic selection of viewpoints that should show suspicious surface regions to simplify the data analysis.

6. Comparative Stress Tensor Visualization

This section comprises four glyph designs to depict stress tensors that fulfill Req. 1, Req. 2 and Req. 3. Furthermore, we give detailed explanations about the vessel and the blood flow visualization that can be combined with the glyph depictions (see Req. 4).

6.1. Vessel and Blood Flow Visualization

For the visualization of the vessel wall with wall thickness and the internal blood flow, we follow the approach by Meuschke et al. [MVB’17]. The user can select two scalar fields that should be visualized simultaneously. The first parameter is depicted using a discretized cool-to-warm color scale. For the second scalar field, the user has to set a threshold. Depending on the parameter, the values are either shown if they are greater or lower than the threshold. The parameter is depicted with a hatching field. The strength of the hatching strokes corresponds directly to the scalar values.

The blood flow, represented by path lines, is visualized with arrow-based glyphs illustrating the flow velocity [LGV’16]. Therefore, we constructed view-aligned quads on the GPU. For a current point in time, the path line is shown with a specific offset. The length of the depicted path line part corresponds to the local flow speed. An additional scalar quantity of the flow, e.g., the pressure, can be depicted by a dark green to light yellow colormap.

6.2. Glyph Distribution

We use the surface vertices as candidates for the glyph placements. The illustration of well-sized glyphs on all vertex points simultaneously, independent from the zoom level of the camera, would lead to visual clutter. Thus, we ensure that neighboring glyphs have a minimal distance. During the interaction with the surface model, the distance may vary depending on the zoom level. This leads to a details-on-demand approach, such that glyphs are only drawn at certain regions, and whenever the user zooms into a region, more glyphs appear.

For this, a hierarchy of regions on the surface is generated. First, we start with an initial vertex \( i_1 \) and determine the geodesic distance following the method by Crane et al. [CWW13]. The heat flow is integrated by solving the heat equation \( \dot{u} = -\Delta u \) at \( i_1 \) for a fixed point in time \( t \) using an implicit Euler integration. Then, the normalized gradient of the heat flow \( u \) is determined: \( X = -V u / |V u| \). Last, the Poisson equation \( \Delta \phi = V \cdot X \) is solved, which yields the geodesic distance from \( i_1 \). Thus, the first level of the hierarchy consists of the vertex \( i_1 \) and this vertex is assigned to all the other vertices, which together form a region (in the first level the whole mesh). Afterwards, we search for the vertex \( i_2 \) with \( i_2 = \arg \max_{i \in V} \phi_i(i) \), where \( V \) encompasses all vertices of the mesh. Then, we determine the geodesic distances \( \phi_2 \) starting from vertex \( i_2 \). A vertex \( j \) is assigned to the vertex \( i_2 \), if \( \phi_2(j) < \phi_i(j) \), thus the regions of both vertices are defined by means of the Voronoi area. We repeat this algorithm until the last level of the hierarchy consists of all vertices of the mesh. Note that in step \( k \) the following vertex \( i_k \) is added:

\[
i_k = \arg \max_{i \in V} \min(\phi_1(i), \phi_2(i), \ldots, \phi_{k-1}(i)).
\]

The user can set the number of vertices \( N \) where a glyph appears. As a presetting, we used 100,120,140,160,180,200,250 vertices for different zoom levels. Moreover, we append six levels including the 250 representatives and each 60th, 40th, 20th, 10th, 3rd and each vertex, respectively. Fig. 3 presents the glyph placement for different zoom levels \( L \) including the according surface regions. The glyphs’ size is adopted to the current zoom level as in [SPGL’14].

6.3. Glyph Representations

This section presents different techniques to illustrate a stress tensor. For the remainder of this section, we use the following notation: A stress tensor \( T \) is a real-valued symmetric 3 \( \times \) 3 matrix: \( T \in \text{Sym}(3) \). The eigenvectors \( v_1, v_2, v_3 \) are ordered such that for the corresponding eigenvalues the property \( \lambda_1 \geq \lambda_2 \geq \lambda_3 \) holds. Then, the eigenvectors express the principal stress directions (PSC)is of
\( T \) and the eigenvalues the corresponding stress values. Note as \( T \) is symmetric, the eigenvalues are real-valued. Furthermore, the eigenvectors are mutually perpendicular. In addition, \( \lambda_3 \) points in direction of the surface normal, see Sec. 4. In the following, we describe how we visualize the eigenvectors and the eigenvalues.

### 6.3.1. 3D Superquadric Visualization

The conventional method to depict tensors is SQs [Kin04]. Therefore, the eigenvalues are transformed into the Westin’s measure [WME99]:

\[
    c_l = \frac{\lambda_l - \lambda_2}{\lambda_1 + \lambda_2 + \lambda_3}, \quad c_p = \frac{2 \cdot (\lambda_3 - \lambda_2)}{\lambda_1 + \lambda_2 + \lambda_3}, \quad c_s = \frac{3 \lambda_3}{\lambda_1 + \lambda_2 + \lambda_3}. \tag{1}
\]

Afterwards, the glyphs are defined by:

\[
    \mathbf{q}_l(\theta, \phi) = \begin{pmatrix} \cos^a(\theta) \sin^b(\phi) \\ \sin^a(\theta) \sin^b(\phi) \\ \cos^b(\phi) \end{pmatrix}, \quad \mathbf{q}_s(\theta, \phi) = \begin{pmatrix} \cos^a(\phi) \\ -\sin^a(\phi) \sin^b(\phi) \\ \cos^b(\phi) \end{pmatrix}, \tag{2}
\]

with \( 0 \leq \theta \leq 2\pi, 0 \leq \phi \leq \pi, \) and \( x^a := \text{sgn}(x)|x|^a. \) If \( c_l \geq c_p \), then \( \alpha = (1 - c_p)^T, \beta = (1 - c_l)^T \) and \( \mathbf{q}_l \) is used. Otherwise, \( \alpha = (1 - c_l)^T, \beta = (1 - c_p)^T \) and \( \mathbf{q}_s \) is used. We use \( \gamma = 3. \) Then, the glyph is oriented such that it directs along \( \mathbf{v}_1 \), where \( \mathbf{v}_1 \) and \( \mathbf{v}_2 \) are scaled with \( \lambda_1 \) and \( \lambda_2 \), respectively. To show tensor differences between the vessel walls, we visualized the glyphs of both walls transparent, see Fig. 1(b). Tensors of the inner wall are depicted in yellow, whereas bluish glyphs show outer wall tensors. To visualize the PSDs even if the \( \lambda_4 \) are similar, we colored the ends in dark blue/dark yellow for the outer/inner wall respectively, see Fig. 4.

### 6.3.2. 2D Kite Visualization

Inspired by Schultz et al. [SK10], we use kite-shaped 2D glyphs to represent \( \mathbf{v}_1 \) and \( \mathbf{v}_2 \) and their corresponding eigenvalues. At every vertex where a glyph should be placed, cf. Sec. 6.2, we generate a view-aligned quad oriented with the surface normal. This quad is equipped with a coordinate system \( x, y \in [-1, 1] \), whereas the \( x \)-axis aligns with the direction of \( \mathbf{v}_1 \). Note that we scaled the axes with their stress values. Then, two glyphs are drawn on the quad, see Fig. 1(c). The first glyph shows the direction and the stress quantities of the inner vessel wall. Thus, for the first tensor we draw:

\[
    C_{\text{inner}} = \begin{cases} 
    (0.94, 0.92, 0.35) & \text{if } 0 \leq x^2 + y^2 < 0.9 \\
    (0.8, 0.79, 0.07) & \text{if } 0.9 < x^2 + y^2 \leq 1 \\
    \text{discard} & \text{otherwise}
    \end{cases}, \tag{6}
\]

cf. Sec. 6.3.1. We set \( \gamma = 0.6. \) The first condition describes the filled color and the second one the boundary. For the outer tensor, we use Eq. 6, but with the filled color \((0.2, 0.3, 0.5)\) and the boundary color \((0.0, 0.5)\). Note that the second glyph is paint over the first tensor. However, to avoid occlusion, the boundary of the first tensor is always drawn. After the first evaluation, cf. Sec 7, the domain expert PI stated that the kite glyphs show the PSDs if the zoom level is appropriate. To show the PSDs even if the zoom level is quite low, we design a glyph that transforms an arrow glyph to a kite glyph based on the distance to the glyph itself. For this, we define \( z \in [0, 1] \) as the zoom level. Based on the coordinate axes \( x, y \in [-1, 1] \) of the view-aligned quad, we define an allowed \( y \)-range:
This yields the simple Euler integration, where at the first iteration step the current triangle to determine the subsequent iteration steps. For every step, we test if $|y| \leq y_{allow}$ holds. If this is the case the fragment is drawn otherwise it is discarded. Additionally, we draw a border around the glyph with the color of the border if $|x| \geq (\frac{1}{2} - (1 - z), 0.3)$. Fig. 5 shows the transition of the arrow to the kite depending on the zoom level.

### Implementation

Similar to the SQs, we use the generated plane to construct the glyphs. First, we align the plane to the normal vector of the triangle. Then, we use the PSDs as a coordinate system on the plane. This yields texture coordinates which can be used in the fragment shader to apply Eq. 6.

#### 6.3.3. 2D Streamline-based Glyph Visualization

Another approach to visualize the tensor is to determine streamlines (SL) on the surface. Fig. 6 shows the streamline visualization based on the ConFIS method. For every generated quad, we assign coordinates which represent the width (orthogonal to the first PSD) in the range $[0, 1]$. The quad is then visualized with a violin plot. The radius of the outer violin plot represents $\lambda_1$ and the radius of the inner violin is determined with $\lambda_2$. We enable the possibility to depict streamlines for the inner and outer vessel wall simultaneously, see Fig. 1(d). Thereby, yellow and purple are used to depict the eigenvalues on the inner wall and blue and green show $\lambda_1$ and $\lambda_2$ on the outer wall.

### Implementation

The streamlines are computed on the GPU. For every triangle $\triangle$ a seed point is generated at the midpoint $p_{mid}$. At $p_{mid}$, we use the average of the PSDs of the vertices belonging to this triangle to determine the mean PSD $v_1$. Afterwards, the point is traced along the mean direction by $p_{new} = p_{old} + h\cdot v(p_{old})$, with a step size $h$ of half of the mean of all edge lengths on the surface mesh. This yields the simple Euler integration, where at the first iteration step $p_{old} = p_{0}$. $v(p)$ denotes the barycentric interpolation of $v_1$ at the triangle vertices at position $p$. After every step, we check if the point leaves the triangle. If this is the case, we update the current triangle to determine the subsequent iteration steps. Additionally to the position of the streamline points, we store the triangle ID as well. This enables to determine the current triangle normal, which is necessary for the generation of the quads on the surface [LLPH15]. Additionally texture coordinates can be used to employ the violin plot in the fragment shader.

#### 6.3.4. 2D Scatterplot Glyph Visualization

We propose different glyphs to encode the PSDs. However, the presented methods indicate the local stress values, but lack to show the direction distributions over a region (see Fig. 5). Thus, we developed a scatterplot-based glyph to display the PSDs at the representative $r_i$, see Fig. 7. This gives an overview about the stress direction distribution according to the vertices in that region $\mathcal{R}_i$. The creation of the scatter-plot (SP) is calculated as follows: first, the representative spans the tangential space, which is given by the normal vector $n_i$, at this vertex $r_i$. Second, the first two PSDs yield an orthogonal coordinate system on the tangent space. Finally, the PSDs of the other vertices $j \in \mathcal{R}_i$, which correspond to the region $\mathcal{R}_i$ are determined and mapped to the tangent space of $r_i$. Based on the discussion with the domain experts, we used a simple projection of the PSDs onto the tangent space. For this, we define the length-preserving projection operator as:

$$P_n(v) = \frac{(Id - n \cdot n^T)}{\|([Id - n \cdot n^T]v)\|} \cdot v,$$

where $Id$ denotes the identity matrix and $\|\cdot\|$ the Euclidean length. $P_n(v)$ projects the vector $v$ to the tangent space of the representative $r_i$. This mapping does not preserve perpendicularity of the two PSDs, but the result is more intuitive for our domain experts as it supports the idea to look from above onto the region $\mathcal{R}_i$ and observe the directions. After the projection, we transformed the coordinates $(x', y', z')$ to $(x, y)$ to the coordinate system of the representative tangent space. Then, we draw small circles on the tangent space at the coordinates $(x, y)$ and $(-y, x)$ as the sign is not uniquely determined. This could be done for PSDs on the inner and outer vessel wall, see Fig 1(c). Additionally, we draw the coordinate axes, which correspond to the PSDs of the representative vertex, where black is used for the first PSD. Then, equidistant circles are drawn for a better estimation of the stress values. The background color changes linearly from fully transparent to orange. This yields the possibility that the user gains insight into the color-coded scalar field on the surface at the current representative vertex. In the following, we give an explanation about the implementation.

#### Implementation

The SP is constructed as a view-aligned quad. To show the PSDs of the vertices at the region $\mathcal{R}_i$, we use the OpenGL extension SHADER_IMAGE_LOAD_STORE. As we use maximal 250 regions, we constructed an image of size $(25 \cdot \text{res}) \times (10 \cdot \text{res})$. Thus, we divide the image in 250 regions with a size of $\text{res} \times \text{res}$. We used $\text{res} = 200$. Every vertex $j$ inherits its PSDs $v_{1j}, v_{2j}$, their assigned region $\mathcal{R}_i$, and the representation’s information like its PSDs and the normal. Every vertex determines first the region in...
Afterwards, the projection operator $P_{\mathcal{R}}(v)$ is applied to $v^j_1$, $v^j_2$, which yields the projected PSDs. A coordinate transformation leads to the coordinates $(x, y)$ and $(-x, -y)$ for one eigenvector (so four coordinates are determined for both PSDs). Finally, the OpenGL function `imagestore()` is applied to draw the directions at the corresponding image position. To display the result, the view-aligned quad of the representative $r_i$ loads the fragment’s color at the corresponding image position, which is given by the region $\mathcal{R}$.

### 6.4. Automatic Viewpoint Selection

To facilitate the time-dependent data analysis and to reduce the interaction to find rupture-prone areas, we integrate an automatic viewpoint selection (seeReq. 6). Therefore, we use the approach by Meuschke et al. [MEB+17]. They calculated optimized views on the surface of cerebral aneurysms depending on two user-selected scalar fields, e.g., the WSS and pressure. Their aim is to find views that maximize the size of the visible aneurysm surface and to show significant areas according to the selected parameters. The viewpoint selection is modeled as an optimization problem, where both criteria are summed up in a target function that is solved by the gradient ascent method. To ensure that interesting regions are in the center of the screen, an additional binomial filter is used. The screen is divided in $n \times n$ subimages and a factor $B_{ij} = \binom{n-1}{i-j} \cdot \binom{n-1}{j}$ is assigned to each subimage $(i, j)$, with $n = 5$. The filter weights the result such that interesting regions in the viewpoint center are emphasized. We use this method to provide additional views on the aneurysm surface that show rupture-prone areas related to stress data. Thus, for the first parameter the $\lambda_1$ field is used and the second parameter can be set by the user. By default, the wall deformation is chosen. For every time-step, we use the first three best views that are presented in smaller widgets additional to the main renderer context, see Fig. 1(a). The widgets can be selected, which loads the corresponding view in the main context for a further exploration.

### 7. Evaluation

To assess the quality of the tensor glyphs, we conducted two evaluations. The first one was an informal evaluation with four domain experts, one physician P1 with an experience of 13 years, two CFD experts P2, P3 with three and six years of experience, respectively, and one visualization expert P4 with a strong focus on cerebral aneurysms, with five years of experience. The second evaluation, was conducted with 60 probands (23 female, 37 male; age range from 20 to 48). Among them were 44 students from computer science and medical engineering. 12 researchers with background in medical visualization, and the four domain experts. They had to answer a questionnaire that assesses how well certain quantities with respect to the glyph designs can be estimated. Sec. 7.1 covers the expert evaluation, and Sec. 7.2 deals with the user study.

#### 7.1. Informal Expert Evaluation

The evaluation was conducted in two steps:

1. A questionnaire that inquires the importance of stress tensors on aneurysms, the glyph designs, and the viewpoint selection. The first step is necessary for the experts to familiarize themselves with the framework. Afterwards, the experts answered the questionnaire. All questions were divided in different categories, which were answered using a five-point Likert scale ($-1$, $0$, $1$, $2$, $3$). For the analysis of the Likert score, we provide the number $S(i)$ of experts who chose the individual scale.

#### Evaluation of intracranial aneurysm. In this category, we inquired about the importance to estimate the stress tensor, the combination of stress tensors with scalar and vectorial data, and the visualization of stress tensors during the cardiac cycle. The first three questions were rated with $S(++) = 2$ and $S(+) = 2$. P1 stated “[...] the estimation is quite important for the analysis of the stress during the cardiac cycle for potential rupture risk.” Only question four was rated as highly important by all users ($S(++) = 4$). P2 and P3 stated that vessel regions, where strong local changes of the PSDs occur during the cardiac cycle could be rupture-prone areas.

#### Evaluation of the tensor glyphs. We asked for all glyphs (superquadric=SQ, kite=K, streamlines=SL, scatterplot=SP), which can be used to:

a) estimate the PSDs of the outer wall,
b) estimate the main stress values of the outer wall,
c) estimate the PSDs simultaneously of the outer and inner wall,
d) estimate the main stress values simultaneously of both walls.

To estimate the PSDs, the participants valued SQ with $S(++) = 4$ and K with $S(++) = 2$, $S(+) = 2$ as most suited. P3 argued that “[...] the kites give me a clear insight to the data.” The SL ($S(++) = 3$; $S(+) = 1$) and SP ($S(++) = 3$; $S(+) = 1$; $S(-) = 1$) are rated more divergent. For question b), K was rated as the best glyph with $S(++) = 2$ and $S(+) = 2$. Also SQ is suitable to determine the PSDs ($S(++) = 2$; $S(+) = 1$; $S(-) = 1$). Similar to question a), SL ($S(++) = 2$; $S(+) = 2$) and SP ($S(+) = 3$; $S(-) = 1$) are evaluated more different. For question c), K and SL were the favorable choice that were valued in the same way with $S(++) = 2$, $S(+) = 1$ and $S(-) = 1$. SQ was also perceived as very appropriate to assess the PSDs ($S(++) = 3$; $S(+) = 1$), whereas SP was least suitable for this task. All experts confirmed that SQ and K are very appropriate to assess the local main stress values ($S(++) = 1$; $S(+) = 2$; $S(-) = 1$). In contrast, SL was perceived as least suited ($S(+) = 4$), whereas SP was rated with $S(++) = 1$, $S(+) = 1$ and $S(-) = 2$.

#### Evaluation of the viewpoints. We asked if the preselection of the viewpoints gives a fast overview about the scene. Furthermore, if the viewpoints are meaningful and if the viewpoint selection with a following camera rotation supports the navigation. All experts answered the first two questions with $S(++) = 3$ and $S(+) = 1$. The third question was rated with $S(++) = 2$ and $S(+) = 2$. P1 argued: “[...] the viewpoints gave me a quick overview about the data and allows an easy navigation.” P4 wished to have a more smooth camera transition between selecting two optimal views successively.

#### Conclusion. All experts were able to use the tool without major problems and would employ it for the analysis of cerebral stress...
tensors. After a short explanation of the exploration and navigation features, the participants could find suspicious surface regions.

![Figure 8: Results of the user study.](image)

### 7.2. Evaluation of the Estimation of the Glyph Tensors

The second evaluation analyzed how well the tensor glyphs served to estimate certain quantities. We prepared an additional questionnaire with screenshots where no framework was involved. The 60 participants were shown different scenes with different glyph types. First, we showed them randomly ordered scenes where we asked whether the stress value is higher for the outer or for the inner vessel wall. Then, we analyzed the results to get a first impression which glyph might be better for the visualization of this task. We did this with four tasks, where the probands should assess:

1. whether the stress is higher for the inner or the outer vessel wall,
2. how much the second stress value takes as a percentage of the first one,
3. what angle difference occurs between the PSDs on both walls,
4. what angle difference occurs between the median main direction and the main direction of the region representative.

The result of these tasks can be found in Fig. 8. For the first task, we compare SQ with K, where we pretend different levels of difficulty with 10%, 30%, 60% and 80% difference between the inner \( \lambda_1 \) and outer \( \lambda_1 \) value, see Fig. 8(a). Here, we excluded SL, because of possible occlusions between lines for the inner and outer wall. For more similar glyphs on both walls (10%, 30%, 60% size difference), K is more suitable to localize the higher stress value. Compared to SQ, twice as many answers were correct for K. For huge differences in size (80%), the results were equally well. The results of the second task are depicted in Fig. 8(b) by using box plots. These display the deviations of the users given values to the correct value. Therefore, we subtracted the users’ values by the ground truth. The first and third quartiles are depicted in orange and blue as well as the minimum and maximum. Here, the probands should approximate the relationship between \( \lambda_1 \) and \( \lambda_2 \) for SQ, K and SL on the inner wall based on four levels of difficulty. Again SQ and K lead to similar good results. If the second value is comparatively small (15%, 30%) K has a slightly lower third quartile in both cases.

For 65%, K dominates clearly over SQ and SL in both quartiles. For 87% K and SL give similar results, but are better suited as SQ. The third task analyzes the angle difference between the PSDs of the inner and outer vessel wall, see Fig 8(c). For this task, SQ and K were compared, whereas SL cannot be used similar to the first task. K dominates clearly over SQ in all cases. In the fourth task, SQ and K are evaluated against SP to determine the angle difference of the median main direction to the main direction of the representative within a given surface region. Therefore, in all shown pictures a coordinate system within a surface region was given, where the x-axis represents the main direction of the representative. SP is clearly better suited for this task compared to SQ and K. The distribution of angle values is smaller than for SQ and K and the third quartile values are close to the exact value, see Fig 8(d).

### 8. Discussion

We evaluate four glyph-based techniques to analyze cerebral stress tensor data comparatively. During the evaluation, it was transpired that the glyphs are differently suitable depending on the task. This was further confirmed in the user study. To get a first overview...
of the PSDs on both walls, SL are very appropriate. The experts were interested in exploring the internal flow together with SL, see Fig. 9(a). They stated that this depiction could be used to find correlations between rupture-relevant flow patterns such as vortices and suspicious tensor configurations. Moreover, SL can be used to uncover opposite regions on the aneurysm with a very different behavior of the PSDs that should be further explored, see. Fig. 10. For a more detailed exploration, other techniques such as SQ and K are more suited, because of attending occlusion problems between streamlines for both walls. We presented the kites in a previous interview to our experts. However, they described the determination of the PSD as very difficult, if the camera has a greater distance to the surface. We improved this by including the transition from the arrow glyph to the kite and repeat our expert evaluation, which leads to the Likert assessments, presented in Sec. 7.1. By using the kites, the experts were able to detect regions with a strong local change of the PSD immediately as possible rupture-prone areas, see Fig. 9(b). For the rotating kites, they used the metaphor of a compass, which was described as very intuitive to perceive strong directional changes. The suitability of the kites to represent percentage and angular differences was also confirmed in the user study.

For the estimation of the stress values, the kites are most suited, because their shape communicates the stress magnitudes immediately. In contrast, the experts described this task based on the line width as more difficult. This was also confirmed in the user study, where K, followed by SQ, led to the best results. SP is also less appropriate for a local approximation of the PSDs. However, it is most suited to analyze the tensor data quantitatively within a surface region, whereas the other techniques allow a qualitative analysis. SP allows a quantification of the homogeneity of the PSDs. Strong inhomogeneous regions paired with suspicious scalar values such as a high WSS could be a rupture-prone criterion, see. Fig. 11. In this context, P1 and P2 asked for the possibility to add landmarks with notes that would facilitate the collaboration between domain experts. Moreover, P3 wished for a drawing feature that annotates various regions with different color and text.

**Further Applications.** Besides the rupture risk and treatment assessment, our tool may also be helpful to explore the predicted wall mechanics and blood flow after different treatment options, such as coiling and stenting. Another important application field is research and education of students. Moreover, CFD and FSI play an essential role also in other vascular structures, such as the aorta, to better understand cardiovascular diseases. A common disease of the aorta is an abdominal aortic aneurysm. The amount of vessel wall deformation is even larger there and the solution presented here would probably be useful in these applications as well.

### 9. Conclusion and Future Work

We presented a framework to analyze cerebral aneurysms for a potential rupture risk assessment. For the first time, we integrated stress tensors in combination with wall thickness and blood flow data for the analysis. So far, there is no accepted factor that describes the likelihood of a rupture. In addition to morphological parameters, CFD-based parameters such as WSS also appear to play a role. Calculating this factor is very difficult, since clinicians evaluate the risk of rupture differently based on their experience. In this context, the wall structure is still not very much discussed clinically. Our tool is intended to enable the introduction of the concept of wall stress into clinical discussions by providing novel glyph visualizations of the complex tensor information. We support the research to eventually establish such a general risk factor. As soon as an accepted criterion exists, this can be added to our framework. To avoid open-ended visualization exploration, we conferred with clinical experts and carried out an extensive evaluation to find out which requirements had to be met and which visualizations could be used to support the exploration and rupture-risk assessment.

Although our tool was rated with positive feedback, we identified several aspects that might be improved. First, for the kites and scatterplots we would like to incorporate the approach by Rocha et al. [RASS17]. Their method draws 2D glyphs directly on the surface instead of using a quad. Moreover, we want to provide a more detailed exploration of aneurysm wall layers. Besides the inner and outer vessel wall, there are intermediate layers in the FSI model with corresponding information such as stress tensors or WSS. To analyze these additional layers and their characteristics, we would like to integrate interactive cutting planes through the aneurysm geometry. Furthermore, we want to incorporate an automatic report generation that summarizes the optimal views and all interesting findings such as presented in Fig. 9, Fig. 10 and Fig. 11. This would further reduce the manual exploration effort and would facilitate the collaborative rupture-risk assessment. Another point for future work would be a larger study with several clinical experts that comprises and discusses the integration of the tool into the clinical workflow. Our long term goal is to contribute to a comprehensive risk assessment that includes also inflammatory factors and real measured wall thickness values, e.g., an assessment of the whole vessel segment where the aneurysm occurs. Basic research clearly indicates that these affect the rupture risk, however in clinical practice such information cannot be acquired so far.

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### References


