Comparison of Divergence-Free Filters for Cardiac 4D PC-MRI Data

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Abstract. 4D PC-MRI enables the measurement of time-resolved blood flow directions within a 3D volume. These data facilitate a comprehensive qualitative and quantitative analysis. However, noise is introduced, e.g., due to inhomogeneous magnetic field gradients.
Blood is commonly assumed as a non-Newtonian fluid, thus, incompressible, and divergence should be zero. Divergence-free filters enforce this model assumption and have been shown to improve data quality.
In this paper, we compare binomial smoothing and three of these techniques: The finite difference method (FDM) [1], divergence-free radial basis functions (DFRBF) [2] and divergence-free wavelets (DFW) [3]. The results show that average and maximum velocities tend to decrease, while average line lengths tend to increase slightly. We recommend FDM or DFW divergence-free filtering as an optional pre-processing step in 4D PC-MRI processing pipelines, as they have feasible computation times of few seconds.

1 Introduction

4D phase-contrast magnetic resonance imaging (4D PC-MRI) [4] allows to acquire blood flow information as a 3D+time velocity vector field. Unfortunately, the data are prone to noise for various reasons. Proper pre-processing is essential to improve both subsequent qualitative and quantitative data analysis. Yet, simple image smoothing methods do not provide a sufficient correction. Therefore, customized methods were developed for each type of noise. For instance, the expected maximum velocity is a pre-scan parameter that has to be estimated based on experience and literature. If chosen too low, image values may flip and blood seemingly runs in the opposite direction (called *phase wrap*). If chosen too high, the measured vectors' accuracy and angular resolution suffers. Another cause for noise are inhomogeneous magnetic field gradients.

Blood is typically modeled as non-Newtonian, incompressible, laminar fluid. Divergence should be zero and a fluid element's density constant over time. However, noise in the data causes that the obtained divergence is non-zero. A specialized group of filters was established for 4D PC-MRI named *divergence-free filters* [5]. As the name suggests, they try to enforce this model assumption, which results in a smoother, theoretically more correct flow field.

In this work, we assess the results of three selected methods while using simple binomial smoothing as reference. The finite difference method (FDM) [1] reduces noise by projecting the data to a divergence-free vector field. The projection is reduced to a 7-point stencil Laplacian problem (two points on x, y and z plus the center) and is solved with a fast Poisson solver using fast Fourier transformations. Divergence-free radial basis functions (DFRBF) [2] employs a combination of normalized convolution and divergence-free radial basis functions in an iterative least-squares algorithm [6]. Divergence-free wavelets (DFW) [3] propose a soft divergence-free enforcement since it might be non-zero at the vessel boundaries due to partial volume effects.

Our comparison of 15 diverse datasets is based on both quantitative and qualitative criteria. We evaluate resulting pathlines according to different criteria, such as their length. Moreover, we assess measures, e.g., average velocities, for measuring planes in the vessels' cross-sections and perform side-by-side comparisons of the vector fields. Our results suggest that divergence-free filters perform better than binomial smoothing and might be a useful addition in a corresponding 4D PC-MRI pre-processing pipeline.

2 Methods

2.1 Data Acquisition and Pre-Processing

Our 15 datasets were obtained with a 3 T Magnetom Verio MR at the Heart Center in Leipzig, a hospital specialized in diagnosis and treatment of heart diseases. The data comprise both healthy volunteers as well as patients with different cardiovascular diseases, such as aneurysms and aortic valve defects. The image sizes and scales are about $140 \times 190 \times 15$–70 ($1.8 \times 1.8 \times 1.8$–$3.5$ mm) with 15–20 temporal positions (40–60 ms). The expected maximum velocity was chosen between 1.5–3.0 m/s, depending on the patient-specific situation, and phase unwrapping was performed [7]. A vessel surface is extracted from a binary segmentation via marching cubes and then smoothed. Centerlines were extracted [8] with the Vascular Modeling ToolKit (VMTK). Blood flow-representing pathlines are integrated using Runge-Kutta-4. Köhler et al. [5] provide a comprehensive overview about the general 4D PC-MRI data processing pipeline.

2.2 Implementation and Parameters

All methods were implemented in C++. The three divergence-free filters are based on MATLAB code provided by Ong et al. [3]. OpenMP was used to parallelize the computation of individual time steps (each algorithm considers one temporal position at a time). Optimization was set to -O3.

For binomial smoothing (Binom) we used an isotropic kernel size of 3. Analogous to the divergence-free filters, each temporal position is processed separately. The finite difference method (FDM) requires no further settings. *SureShrink* [9] was used as threshold for Divergence-free Wavelets (DFW) with $spin = 2$, both as suggested by the authors [3]. For divergence-free radial basis functions (DFRBF) we used an isotropic convolution kernel of size 3. This rather small size limits the smoothing effect that comes with RBF. In our examples, convergence was observed experimentally at about 20 iterations, which we use as default.

2.3 Comparison

We calculate all of the criteria below for each dataset in every configuration (original, different filterings) and then calculate ratios where the original is the reference. The ratios indicate whether the corresponding criterion decreases (values $< 100\,\%$) or increases (values $> 100\,\%$) after filtering. The ratios' distribution (one ratio per dataset per filtering) will be presented as box plots. The criteria were inspired by divergence-free papers [1,2,3].

Our first employed measure is the *average divergence* within the vessel segmentation. For this, we manually chose the time step that represents peak systole (when the blood is pumped).

Four equidistant measuring planes were placed, starting inside the ascending and ending in the descending aorta, where higher and lower velocities are expected, respectively. Besides a *qualitative comparison* of the in-plane vector fields, we evaluated the *average* and *maximum velocity vector magnitudes*. To increase robustness, we use the $95\,\%$ quantile as maximum.

We integrate one pathline for each voxel of the vessel segmentation in 3 temporal positions: peak systole and its predecessor and successor. For these pathlines, we calculate their *absolute length* as accumulation of Euclidean distances between subsequent line segments. The temporal components are ignored. Also, we calculate their *relative length* by projecting all pathline points onto the vessel's centerline and then determining the centerline's arc length between the two projected points closest to the beginning and end of the centerline. This measure resembles the distance of end points while taking into account the curved vessel as domain. If there is a significant deviation of absolute and relative line length, this is an implicit indicator for increased curvature, e.g., due to vortex flow. The last measure reuses the measuring planes. It describes how many *pathlines connect the first and the last plane*. An increase of this measure indicates that less lines prematurely abort because they run out of the segmentation due to noisy flow directions.

3 Results and Discussion

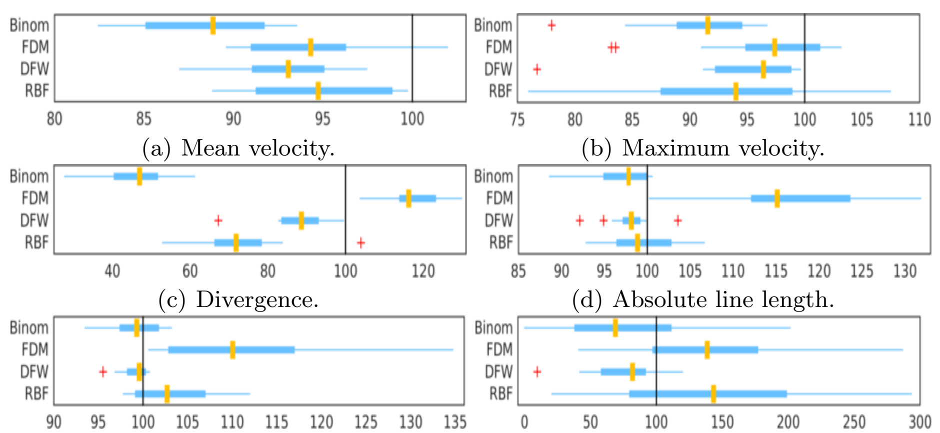
This section starts with a performance assessment of the employed divergence-free methods. We proceed by comparing results of 15 datasets according to the previously described criteria. Fig. 1 illustrates the ratios how each criterion increases or decreases relative to the unfiltered dataset.

The tests were performed on an Intel i5-6400 quad core with 3.4 GHz. Generally, computational effort depends on the image size. DFRBF additionally depends on the number of iterations. DFW computation time increases with higher spin values. On average, FDM was performed in 1–3 s, DFW in 15–45 s, and DFRBF in 5–15 min. We consider up to 1 min as feasible for integration in a processing pipeline of a corresponding evaluation tool. Thus, DFRBF is not appropriate in this respect.

Fig. 2 shows an exemplary comparison of resulting divergence fields. Binomial smoothing consistently lowers the divergence as velocities and their derivatives become smaller. To our surprise, FDM increases the divergence by median +16.3 % (see Fig. 1(c)). DFW and DFRBF both have decreased the divergence for every dataset by median −11.3 % and −28.1 %, respectively. The comparably strong decrease of DFRBF might be due to the general smoothing that comes with using RBF. For DFM and DFW the decrease is not as strong as we expected. The high remaining divergence values at the segmentation boundaries are a known problem in the corresponding papers [2].

The results from Figs. 1(a)–(b) indicate that both the mean and maximum velocities are decreasing for all methods, though, not as strongly as with simple binomial smoothing. There are mostly minor changes up to 10 %. This seems plausible since a certain degree of smoothing comes with application of the filters. The smoothness of DFRBF is comparable to binomial filtering, FDM has the least degree of smoothing and DFW is in between. Fig. 3 shows an exemplary measuring plane. Velocity changes can be crucial, since velocities directly influence quantitative measures, such as net flow volumes, that assess the flow passing a measuring plane.

FDM significantly increases both the pathlines' absolute and relative length up to +36.4 % and +34.7 % maximum and +15.1 % and +10.1 %, respectively (see Figs. 1(d)–(e)). DFW produces approximately the same absolute and relative line lengths as the unfiltered datasets (median −1.9 % and −0.4 %), so does

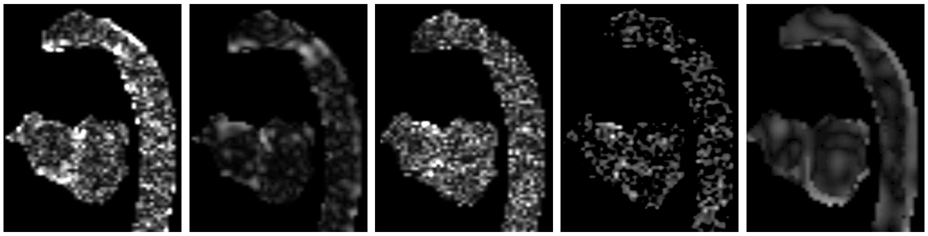


(a) Mean velocity. (b) Maximum velocity.

(c) Divergence. (d) Absolute line length.

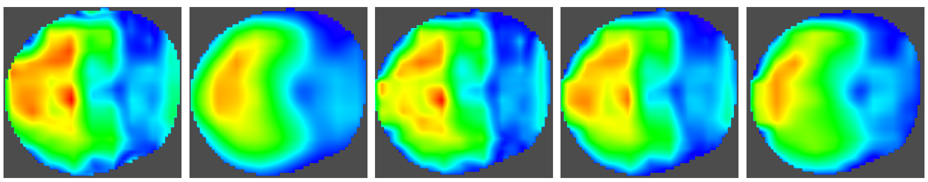
(e) Relative line length. (f) #Lines connecting first and last plane.

Fig. 1. The box plots depict how each criterion (see Sec. 2.3) changes w.r.t. the unfiltered original. Black vertical lines mark the reference at 100 %. Orange lines are median values. Blue boxes are interquartile ranges. Red crosses are outliers.



(a) Original. (b) Binomial. (c) FDM. (d) DFW. (e) DFRBF.

Fig. 2. Divergence field of a healthy volunteers aorta during systole in sagittal orientation. Binomial smoothing (b) decreases, FDM (c) slightly increases, DFW (d) slightly decreases, and DFRBF (e) strongly decreases the divergence field. The divergences at the vessel boundaries remains comparably high (c-e), which is a known behavior in the corresponding papers.



(a) Original. (b) Binom. (c) FDM. (d) DFW. (e) DFRBF.

Fig. 3. Velocities (rainbow color scale) of a measuring plane inside the ascending aorta of a patient during systole. The general velocity distribution is preserved in all approaches, however, the degree of smoothing strongly varies. The DFRBF result is the smoothest, which is confirmed by the highest decline of average velocities.

DFRBF (median $-1.1\,\%$ and $+2.7\,\%$). For the latter two approaches, visual pathline changes are not noticeable. For FDM we could observe situations where laminar pathlines, starting in the ascending aorta, followed the vessel course longer than before, i.e., they reached farther into the descending aorta, which is physiologically expected. This is underlined by the number of lines connecting the first and last measuring plane (see Fig. 1(f)). Here, FDM achieves a median improvement of $+38.5\,\%$. Interestingly, DFRBF even reaches $+43.4\,\%$ improvement although the line lengths were not significantly increased. This could be explained by a straightening of the pathlines. DFW had a decline of $-18.1\,\%$. For all three line-related criteria, DFW is closest to the binomial smoothing result.

4 Conclusion and Future Work

In this work we have evaluated three state-of-the-art divergence-free filters (FDM, DFW, DFRBF) for their influence on the resulting flow field and pathlines. Velocity values decreased up to $10\,\%$ with all methods, which we consider as a reasonable margin. The pathline quality improved in many cases, which was observable via increasing line lengths and more lines being able to follow the vessel course correctly. Divergence did not decrease as strongly as we expected and even increased using FDM. However, standard binomial smoothing, which was used as a reference, produced worse results, e.g., by strongly altering vortex flow patterns. This underlines the value of divergence-free filters.

A drawback of our comparison and 4D PC-MRI data in general is that there is no ground truth. Thus, it is not clear to what extent the differing results are improvements or not. A future work could be to employ 2D PC-MRI, which measures only one slice over time within the vessel, but with a higher spatio-temporal resolution. One could argue that this should produce more accurate results than the same measuring plane in 4D PC-MRI. Hence, if quantification results, e.g., for net flow volumes, come closer to the 2D PC-MRI reference after divergence-free filtering, this should indicate a definite improvement.

We cannot recommend divergence-free filtering as a *mandatory* pre-processing in corresponding 4D PC-MRI evaluation software. Yet, we think it should be provided as an optional step to facilitate getting more experiences with these techniques. With respect to the computation times, FDM ($\leq$ 3 s) and DFW ($\leq$ 45 s) are feasible, whereas DFRBF ($\geq$ 5 min) is not.

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